

SCALAR AND SPINOR PARTICLES WITH LOW BINDING ENERGY IN THE STRONG STATIONARY MAGNETIC FIELD STUDIED BY MEANS OF TWO- AND THREE-DIMENSIONAL MODELS

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On the basis of analytic solutions of Schrodinger and Pauli equations for a uniform magnetic field and a single attractive $\delta(\mathbf{r})$ -potential the equations for the bound one-active electron states are discussed. It is very important that ground electron states in the magnetic field essentially different from the analog state of spin-0 particles that binding energy has been intensively studied at more than forty years ago. We show that binding energy equations for spin-1/2 particles can be obtained without using of a well-known language of boundary conditions in the model of δ -potential that has been developed in pioneering works. Obtained equations are used for the analytical calculation of the energy level displacements, which demonstrate nonlinear dependencies on field intensities. It is shown that in a case of the weak intensity a magnetic field indeed plays a stabilizing role in considering systems. However the strong magnetic field shows the opposite action. We are expected that these properties can be of importance for real quantum mechanical fermionic systems in two- and three-dimensional cases.

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I. FORMULATION OF THE PROBLEM.

The effect of an external electromagnetic field on nonrelativistic charged particles systems (like atoms, ions and atomic nucleuses) has been investigated systematically for a long time (see, for example, [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]). Though this problem has a long history, a set of questions as before requires additional studying. For example, till now the systematic analysis of the bound states of particles with spin-1/2 in the intensive magnetic field is absent. Note that the basic results for the case of spinless particles were obtained by using of analytical solutions in nonperturbative mathematical treatments. As usually the exact solutions of Schrödinger equations with Hamiltonians taking into account a particle bound by short-range potential in the presence of external fields are used. Furthermore there is the rather common opinion on invariably the stabilizing role of magnetic field in decays of quasistationary states [1, 14, 15]. This point of view is caused by the fact that spinor states of electrons in an external electromagnetic field are not usually taken into account in nonrelativistic treatments that is not always adequate [16, 17]. In this paper we treat an essential part of these problems.

There are we consider charged spin-0 and spin-1/2 particles bounded by a short range potential (a δ -potential) and located in the external stationary magnetic field with an arbitrary intensity. Note that the potential of zero-radius is a widely spread approximation for a multi-electronic atom field and especially for a negative ion field [12, 18].

For the particle in a δ -potential and in the magnetic field one can see the energy level displacements. The most adequate instrument for the investigation of such states is the development of the binding energy equation formalism ([4, 12, 19]).

When an electron moves in a uniform magnetic field oriented in z -direction this quantum mechanical system is invariant with respect to z -axis. Then the system becomes essentially two-dimensional in the xy plane. Many physical phenomena occurring in quantum systems of electrically charged fermions, which have the axial symmetry, can be studied effectively by means of the equations of motion in 2+1 dimensions - quantum Hall effect, phenomenon of high-temperature superconductivity, investigation of different

film defects and etc. A number of these effects in constant magnetic fields including the investigation of a certain type of doped two-dimensional semimetals one can find in [20],[21] (treatment of two-dimensional models see also in [22] - [25]). However there are a lot of physical phenomena which as before occur in three-dimensional space. In this paper we investigate the effect of a stationary and uniform magnetic field on localized electron states with regard of $2 + 1$ and $3+1$ dimensions.

The main purpose of this work is to derive equations for the binding energy of a fermion in the field contained an attractive singular potential and a stationary external magnetic field in two- and three-dimensional cases. This treatment is realized by means of the standard quantum mechanical methods using the development of the unknown wave function in a series at the eigenfunctions obtained for the fermionic system in the pure magnetic field. This formalism is principally different from the traditional derivation of wave functions in similar tasks by using the boundary condition typical for the δ -potential [4, 5, 14, 15]. It is very important that our approach let us to develop the consistent investigation of the spin effects arising in the external magnetic field.

The general structure of the paper and its main results we can formulated as follows. In the second Section on the basis of explicit solution for Schrödinger equation the equation for scalar particles with a low binding energy is constructed in an external stationary magnetic field. In the third Section on the basis of the analog analysis of explicit solutions for Pauli equations the expressions for the energy of bound electron states in the δ -potential and in the external magnetic fields are obtained. Finally in the fourth Section the equations for the bound energy of spin-0 and spin-1/2 particles in the presence of a weak and strong magnetic fields is simultaneously discussed because early in similar tasks the spin of particles is not adequate taken into account.

II. A SCALAR PARTICLE IN AN ATTRACTIVE POTENTIAL IN THE PRESENCE OF A UNIFORM MAGNETIC FIELD

Let us consider a charge in a uniform magnetic field \mathbf{B} , which is specified as

$$\mathbf{B} = (0, 0, B) = \nabla \times \mathbf{A}, \quad \mathbf{A} = (-yB, 0, 0). \quad (1)$$

The Schrödinger equation in field (1) has the form

$$i\hbar \frac{\partial}{\partial t} \psi(t, \mathbf{r}) = \mathcal{H} \psi(t, \mathbf{r}), \quad \mathbf{r} = (x, y, z), \quad (2)$$

where the Hamiltonian \mathcal{H} is

$$\mathcal{H} = \frac{1}{2m} \left(-i\hbar \frac{\partial}{\partial x} + \frac{eB}{c} y \right)^2 - \frac{\hbar^2}{2m} \frac{\partial^2}{\partial y^2} - \frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2}. \quad (3)$$

Here m ; e – mass and charge of a particle correspondingly. The wave function of the particle in field (1) has the form [19]

$$\psi_{np_x p_z}(t, \mathbf{r}) = \frac{1}{2} e^{-iE_n t/\hbar} e^{i x p_x/\hbar + i z p_z/\hbar} U_n(Y), \quad (4)$$

where

$$E_n = \hbar\omega \left(n + \frac{1}{2} \right) + \frac{p_z^2}{2m} \quad (5)$$

is the energy spectrum of electron, $\omega = |eB|/mc$, p_x and p_z are the momenta of the electron in the x and z -direction respectively.

The functions

$$U_n(Y) = \frac{1}{(2^n! \pi^{1/2} r_0)^{1/2}} \exp \left(-\frac{(y - y_0)^2}{2r_0^2} \right) H_n \left(\frac{y - y_0}{r_0} \right),$$

are expressed through the Hermite polynomials $H_n(z)$, the integer $n = 0, 1, 2, \dots$ indicates the Landau level number, $r_0 = \sqrt{\hbar c/|eB|} \equiv \sqrt{\hbar/m\omega}$ is the so-called magnetic length (see, for example, [26]) and $y_0 = -cp/eB$.

Now we study a simple solvable model. We consider the motion of an scalar particle in the case of three dimensions in a single attractive $\delta(\mathbf{r})$ potential and in the presence of a uniform magnetic field. Here $\delta(\mathbf{r})$ is the Dirac delta function. In fact, the equation we need solve is the following Schrödinger equation

$$\frac{1}{2m} \left[\left(-i\hbar \frac{\partial}{\partial x} + \frac{eB}{c} y \right)^2 - \hbar^2 \frac{\partial^2}{\partial y^2} - \hbar^2 \frac{\partial^2}{\partial z^2} - \hbar^2 \delta(\mathbf{r}) \right] \Psi_{E'}(\mathbf{r}) = E' \Psi_{E'}(\mathbf{r}). \quad (6)$$

Solutions of Eq.(6) we can take in the form

$$\Psi_{E'}(\mathbf{r}) = \sum_{n,p_x,p_z} C_{E'np_xp_z} \psi_{np_xp_z}(\mathbf{r}) \equiv \sum_{n=0}^{\infty} \int dp_x dp_z C_{E'np_xp_z} \psi_{np_xp_z}(\mathbf{r}), \quad (7)$$

where $\psi_{np_xp_z}(\mathbf{r})$ is the spatial part of the wave functions (4).

Coefficients $C_{E'np_xp_z}$ can easily be calculated and then we obtain the following equation

$$1 = N \sum_{n=0}^{\infty} \int dp_z \frac{1}{n + A}, \quad (8)$$

where N - normalized coefficient is independent on the field and

$$A = \frac{1}{2} - \frac{E}{\hbar\omega} + \frac{p_z^2}{2m\hbar\omega}. \quad (9)$$

Integrating under p_z gives the equation (8) in the form

$$1 = N\pi\sqrt{2m\hbar\omega} \sum_{n=0}^{\infty} \frac{1}{(n + A)^{1/2}}. \quad (10)$$

It is easy to see that Eq.(10) defines implicitly energy of a bound localized electron state in the magnetic field. It should be mentioned that (10) is consistent with analog result of [25], where this equation then has been solved numerically.

However Eq. (10) may be analytically reduced to more simple form. Indeed the summation with respect to n on the right-hand side of Eq.(10) can be performed by using the representation

$$\frac{1}{(n + A + i\varepsilon)^{1/2}} = \frac{e^{-i\frac{\pi}{4}}}{\sqrt{\pi}} \int_0^{\infty} \frac{e^{i(n+A+i\varepsilon)t}}{t^{1/2}} dt. \quad (11)$$

As a result Eq.(10) becomes

$$1 = N_1 \sqrt{\hbar\omega} \frac{e^{-i\frac{3\pi}{4}}}{2\sqrt{\pi}} \int_0^\infty \frac{e^{-i\frac{E'}{\hbar\omega}t}}{t^{1/2} \sin(t/2)} dt, \quad (12)$$

where N_1 - the real constant which is independent on the field. As the required energy

$$E' = -|E'|, \quad (13)$$

must be negative, we can rotate the integration contour by angle $\pi/2$ in the complex plane of t . Thus we have a real expression

$$-1 = N_1 \frac{\sqrt{\hbar\omega}}{2\sqrt{\pi}} \int_0^\infty \frac{e^{-\frac{E}{\hbar\omega}t}}{t^{1/2} \sinh(t/2)} dt, \quad (14)$$

where $E = |E'| \geq 0$. If we eliminate the magnetic field, the (14) takes the form

$$-1 = N_1 \sqrt{\frac{\hbar}{\pi}} \int_0^\infty \frac{e^{-E_0 t/\hbar}}{t^{3/2}} dt, \quad (15)$$

where $E_0 = |E'_0|$ is the absolute value of the bound energy of the particle in the δ -potential without the action of the external field. Subtracting (15) from (14) and removing of integrals divergences in the lower limit by the way of standard regularization procedure we have

$$\int_0^\infty \frac{e^{-E_0 t/\hbar} - e^{-Et/\hbar}}{t^{3/2}} dt = \int_0^\infty \frac{e^{-Et/\hbar}}{t^{3/2}} \left(\frac{a_1 t}{\sinh(a_1 t)} - 1 \right) dt, \quad (16)$$

where $a_1 = \frac{E}{2}$. From (16) it is easily to obtain

$$\sqrt{E} - \sqrt{E_0} = \frac{\sqrt{E}}{2\sqrt{\pi}} \int_0^\infty \frac{e^{-x}}{x^{3/2}} \left(\frac{ax}{\sinh(ax)} - 1 \right) dx, \quad (17)$$

where $a = \frac{\hbar\omega}{2E}$, that is consistent with analog equation obtained by the well-known method using boundary conditions of wave functions in the model of δ -potential [5],[14],[27].

In the weak field limit $\hbar\omega \ll 2E_0$ developing the expansion of the integrand function in (17) we can obtain

$$E = E_0 \left(1 - \frac{1}{48} \frac{\hbar^2 \omega^2}{E_0^2} + \frac{1}{576} \frac{\hbar^4 \omega^4}{E_0^4} \right). \quad (18)$$

Note that the square term in (18) is coincided with analog result of [5].

In order to consider the case of a strong field $\hbar\omega > 2E_0$ we produce the right-hand side of (17) to the analytic form

$$-\frac{1}{\sqrt{a_0}} = \frac{1}{\sqrt{2}} \text{Zeta} \left[\frac{1}{2}, \frac{1}{2} + \frac{1}{2a} \right], \quad (19)$$

where $a_0 = \frac{\hbar\omega}{2E_0}$ and $\text{Zeta}[\nu, p]$ is a generalized Riemann Zeta-function. The range of the validity (19) be found some wider as was supposed initially. In fact for derivation of (14) we assume that $E' \leq 0$, however from (19) it should be that argument of Zeta-function can be continuously reached the values

$$1/2 + 1/2a > 0.$$

This condition gives the limitation of the required bound energy spectrum

$$E' < \frac{\hbar\omega}{2}. \quad (20)$$

The physical meaning of this condition consists of restriction to the continuous spectrum of the scalar particle in the magnetic field by the value (20). Note that after change of variables in (19) that is coincided with the basic equation of work [5] where the case of scalar particles in the magnetic field was considered and analog conclusion about limitation of continuous spectrum has been done.

Expansion of $\text{Zeta}[\nu, p]$ in the limit $p \ll 1$ gives the result

$$\text{Zeta}[1/2, p] = \frac{1}{p^{1/2}} + \text{Zeta}[1/2] - \frac{1}{2} \text{Zeta}[3/2]p + \frac{3}{8} \text{Zeta}[5/2]p^2 + 0[p]^3. \quad (21)$$

After substitution of (21) to (19) we can obtain the equation that explicitly determines the bound state energy in the strong field limit

$$E' = \hbar\omega \left(0.205 - 0.452\sqrt{\frac{E_0}{\hbar\omega}} - 0.367\frac{E_0}{\hbar\omega} \right). \quad (22)$$

It should be emphasized that in the super strong magnetic fields expansion (22) gives the upper limit of binding energy of the scalar particle

$$E' = 0.205 \hbar\omega,$$

which is not contradicted to the condition (20). Furthermore one can see that this limited value is not dependent on the particle energy in the absence of the field, but it is completely determined by the magnetic field intensity.

It is of interest to compare the obtained results with the case of two-dimensional model. The analog of the Eq.(10) in the two-dimensional case takes the form

$$1 = \frac{1}{8\pi} \int_0^\infty \frac{e^{-Et/\hbar\omega}}{\sinh(t/2)} dt, \quad (23)$$

what is coinciding with the corresponding result of the work [23]. However, unlike treatment [23] the regularization procedure we carry out in another way. Thus as early (see (16)) we remove the magnetic field and obtain

$$1 = \frac{1}{4\pi} \int_0^\infty \frac{e^{-E_0 t}}{t} dt. \quad (24)$$

Carrying out a simple calculation which is similar to the described regularization procedure for the three-dimensional case we can write

$$\ln \frac{E}{E_0} = \int_0^\infty \frac{e^{-x}}{x} \left(\frac{ax}{\sinh(ax)} - 1 \right) dx, \quad (25)$$

where as before $a = \hbar\omega/(2E)$. In the weak field limit from (25) we have

$$E = E_0 \left(1 - \frac{\hbar^2 \omega^2}{24 E_0^2} \right). \quad (26)$$

For the successive consideration of the range $\hbar\omega > 2E_0$ at first the integral in the right-hand side of the Eq.(25) should be calculated analytically

$$-\ln\left(\frac{E}{E_0}\right) = \ln(2a) + \Psi\left(\frac{1+a}{2a}\right), \quad (27)$$

where $\Psi(x)$ is a logarithmic derivative of Euler Gamma function. Then we have the basic equation in the two-dimensional model

$$-\ln(2a_0) = \Psi\left(\frac{1}{2} + \frac{1}{2a}\right), \quad (28)$$

where $a_0 = \hbar\omega/(2E_0)$.

In the strong field limit, after evaluation of $\Psi(p)$ function

$$\Psi(p) = -\frac{1}{p} - C + \frac{\pi^2}{6}p + \frac{1}{2}\text{PolyGamma}[2, 1]p^2 + \frac{\pi^4}{90}p^3 + 0[p]^4, \quad (29)$$

where

$$\text{PolyGamma}[n, z] = \Psi^{(n)}(z) = \frac{d^n \Psi(z)}{dz^n},$$

(28) can be written in the form

$$\ln \frac{\hbar\omega}{E_0} - \frac{1}{\frac{1}{2} - \frac{E'}{\hbar\omega}} - C + \frac{\pi^2}{6} \left(\frac{1}{2} - \frac{E'}{\hbar\omega} \right) = 0, \quad (30)$$

where $C = 0.577\dots$ is Euler constant.

The solution of Eq.(30) is explicitly determining the bound state energy can be written as

$$\frac{E'}{\hbar\omega} = \frac{1}{2} - \frac{6(C - \ln(\hbar\omega/E_0)) + \sqrt{24\pi^2 + 36(C - \ln(\hbar\omega/E_0))^2}}{2\pi^2}. \quad (31)$$

In the limit $\ln(\hbar\omega/E_0) \gg 1$ from (31) we have

$$\frac{E'}{\hbar\omega} = \frac{1}{2} - \frac{1}{\ln(\hbar\omega/E_0)} - \frac{C}{\ln^2(\hbar\omega/E_0)} + \frac{\left(\frac{\pi^2}{6} - C\right)}{\ln^3(\hbar\omega/E_0)} + 0[\ln(\hbar\omega/E_0)]^4. \quad (32)$$

Considering the properties of $\Psi(z)$ we see again that expansion (32) is correct under the bound energy $E' = \leq \hbar\omega/2$. Furthermore this limited value as before is not dependent on

the particle energy in the absence of the field. However, there is the essential difference from the three-dimensional case. Indeed, the upper limit of the shifted binding energy level in consider model for the range of supper strong magnetic fields (when large not only ratio $\hbar\omega/E_0$, but also $\ln(\hbar\omega/E_0) \gg 1$) has tendency directly to the boundary of the continuous spectrum.

III. AN ELECTRON IN AN ATTRACTIVE POTENTIAL IN THE PRESENCE OF A UNIFORM MAGNETIC FIELD

It is vary important that with the help of the present approach we can study the spin-effects in the magnetic fields by the same way. The case of spin-1/2 particle can be calculated on the basis of exact solutions of Pauli equation. The Pauli equation in the field (1) has the form

$$i\hbar\frac{\partial}{\partial t}\psi(t, \mathbf{r}) = \mathcal{H}\psi(t, \mathbf{r}), \quad \mathbf{r} = (x, y, z), \quad (33)$$

where the Hamiltonian \mathcal{H} is

$$\mathcal{H} = \frac{1}{2m} \left(-i\hbar\frac{\partial}{\partial x} + \frac{eB}{c}y \right)^2 - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial y^2} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + \mu\sigma_3B. \quad (34)$$

Here $\mu = |e|\hbar/2mc$ is the Bohr magneton, m is the mass of a electron and

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

is the z -component of Pauli matrixes. The last term in (34) describes the interaction of the spin magnetic moment of the electron with the magnetic field. Electron wave function in field (1) has the form

$$\psi_{np_x p_z s}(t, \mathbf{r}) = \frac{1}{2}\psi_{np_x p_z}(t, \mathbf{r}) \begin{pmatrix} 1 + s \\ 1 - s \end{pmatrix}, \quad (35)$$

where $\psi_{np_x p_z}(t, \mathbf{r})$ is the solution of the Schrödinger equation in the field (1) (see(4)),

$$E_{ns} = \hbar\omega \left(n + \frac{1}{2} \right) + \frac{p_z^2}{2m} + s\hbar\omega \frac{1}{2} \quad (36)$$

is the energy spectrum of electron, $\omega = |eB|/mc$, $s = \pm 1$ is conserving spin quantum number, p_x and p_z are the momenta of the electron in the x and z -direction respectively.

It is vary important that ground electron state in the magnetic field essentially different from the analog state of spin-0 particles. At the same time the boundaries of continuous spectra for spinor particles will be differ from scalar one. For example, if continuous spectrum of scalar particle begin from value $E' \geq \hbar\omega/2$, for an electron with spin directed along the magnetic field analog condition is $E' \geq \hbar\omega$. For particle with spin opposite to the magnetic field strength continuous spectrum is beginning from values $E' \geq 0$.

Thus taking into account an interaction of the electron spin magnetic moment with the magnetic field the equation for energy in the three-dimensional case can be written in the form

$$\sqrt{E} - \sqrt{E_0} = \frac{\sqrt{E}}{2\sqrt{\pi}} \int_0^\infty \frac{e^{-t}}{t^{3/2}} \left(\frac{ate^{-sat}}{\sinh(at)} - 1 \right) dt, \quad (37)$$

were $s = \pm 1$ corresponds to spin orientations along and opposite to the direction of the magnetic field respectively. Expend the integral in (37) in the limit $a \ll 1$ we have the equation

$$\sqrt{E} - \sqrt{E_0} = -\frac{s\hbar\omega}{4\sqrt{E}} + \frac{\sqrt{E}}{12} \left(\frac{\hbar\omega}{2E_0} \right)^2. \quad (38)$$

The solution of Eq.(38) has the form

$$\sqrt{\frac{E}{E_0}} = \frac{2 \left(-12E_0 - \sqrt{3} \sqrt{48E_0^4 + s(\hbar^3\omega^3 E_0 - 48\hbar\omega E_0^3)} \right)}{\hbar^2\omega^2 - 48E_0^2}. \quad (39)$$

The expansion (39) in the weak field limit can be written as follows:

$$\frac{E}{E_0} = 1 - s \frac{\hbar\omega}{2E_0} - \frac{1}{48} \frac{\hbar^2\omega^2}{E_0^2}. \quad (40)$$

From (40) it is easy to see that the level of energy $E'_0 = -|E_0|$, existing in δ -potential without any perturbation, for the case $s = 1$ under the action of the magnetic field increases at $\frac{\hbar\omega}{2E_0}$, and for $s = -1$ it falls at $\frac{\hbar\omega}{2E_0}$ in terms of the negative energetic scale. However the depth of energetic levels with respect to the boundaries of the continuous spectra for this two cases are equal among themselves and equal to the case of the spin-0 particles.

Integrating of the right-hand side Eq.(37) we can obtain the equation in the analytical form

$$-\frac{1}{\sqrt{a_0}} = \frac{1}{\sqrt{2}} \text{Zeta} \left[\frac{1}{2}, \frac{1}{2} + \frac{s}{2} + \frac{1}{2a} \right]. \quad (41)$$

In the strong field limit $\hbar\omega > E_0$ one can write the generalized Riemann's Zeta-function in Eq.(41) as follows

$$\frac{1}{\sqrt{2}} \text{Zeta} \left[\frac{1}{2}, \frac{1}{2} + \frac{s}{2} + \frac{1}{2a} \right] = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{\frac{1+s}{2} + \frac{E}{\hbar\omega}}} + \frac{\text{Zeta}[1/2]}{\sqrt{2}} - \frac{\text{Zeta}[3/2]}{2\sqrt{2}} \left(\frac{1+s}{2} + \frac{E}{\hbar\omega} \right). \quad (42)$$

Finally, the Eq.(41) can be written in the following way

$$\frac{1}{\sqrt{2}} + \left(\sqrt{\frac{2E_0}{\hbar\omega}} + \frac{\text{Zeta}[1/2]}{\sqrt{2}} \right) x - \frac{\text{Zeta}[3/2]}{2\sqrt{2}} x^3 = 0, \quad (43)$$

where

$$x = \sqrt{\frac{1+s}{2} + \frac{E}{\hbar\omega}}.$$

Solutions of Eq.(43) for different spin values $s = 0, +1, -1$ are represented as

$$E' = \hbar\omega \left(0.205 + \frac{s}{2} - 0.452 \sqrt{\frac{E_0}{\hbar\omega}} - 0.367 \frac{E_0}{\hbar\omega} \right). \quad (44)$$

Figure 1. presents a comparison of the graphic solutions of Eq.(41) for different values of spin under $E_0 = 1$ and $\hbar\omega = 100E_0$. It is evidently, that approximate solutions (44) give results which are near the explicit points of intersections of left-hand and right-hand sides of the Eq.(41). It should be emphasized that in strong field limit the dependence of

energetic level shifts from the spin is not disappeared. However in all cases displacements of binding energy levels as in the weak field limit are at the equal distances with respect to the boundaries of the continuous spectra.

Let us now consider the two-dimensional case with regard for spin interactions. According to present approach we can write

$$\ln\left(\frac{E}{E_0}\right) = \int_0^\infty \frac{e^{-x}}{x} \left(\frac{axe^{-sax}}{\sinh(ax)} - 1 \right) dx, \quad (45)$$

where the direction of spin particles as before is characterized by $s = \pm 1$. In the range of weak fields from Eq.(45) we have

$$\frac{E}{E_0} = 1 - \frac{s\hbar\omega}{2E_0} - \frac{1}{24} \left(\frac{\hbar\omega}{E_0} \right)^2. \quad (46)$$

For consideration of opposite limit it is clear we must calculate the integrals in the above equation in the analytical form

$$\int_0^\infty \frac{e^{-x}}{x} \left(\frac{axe^{-sax}}{\sinh(ax)} - 1 \right) dx = -2a \frac{(1+s)}{2} - \ln(2a) - \Psi\left(\frac{1}{2a}\right). \quad (47)$$

Then equations for energy displacements for $\hbar\omega > E_0$ will be written as

$$\ln\left(\frac{E}{E_0}\right) = 2a \frac{(1-s)}{2} - \ln(2a) + C - \frac{\pi^2}{12a}. \quad (48)$$

For the case $s = -1$ in the strong field limit ($\ln \frac{\hbar\omega}{E_0} \gg 1$) from Eq.(45) at once may be obtained

$$E' = -\hbar\omega \left(\frac{1}{\ln \frac{\hbar\omega}{E_0}} + \frac{C}{\ln \frac{\hbar\omega}{E_0}} \right). \quad (49)$$

For the opposite spin orientation ($s = 1$) in Eq.(47) we at first must use the recurrent relation for $\Psi(p)$ function

$$\frac{1}{x} + \Psi(x) = \Psi(1+x). \quad (50)$$

After that using the asymptotic expansion for $\Psi(p)$ (see (29)) we also have

$$E' = \hbar\omega \left(1 - \frac{1}{\ln \frac{\hbar\omega}{E_0}} - \frac{C}{(\ln \frac{\hbar\omega}{E_0})^2} \right). \quad (51)$$

It is ease to see that dependence from spin parameters can be interpreted as before in three-dimensional model. The main difference from this case that in the supper strong magnetic field we have convergence of the considered binding energy levels to the boundaries of continuous spectra.

IV. DISCUSSION AND CONCLUSIONS

We have shown that the effect of a magnetic field on localized electron states leads to the number of equations for the binding energy of spin-0 and spin-1/2 particles. Thus, in the weak field limit energy displacements scalar and spinor particles in the case of the three dimensional model can be described by the following expressions

$$\left. \begin{array}{l} s = 0 \\ s = 1 \\ s = -1 \end{array} \quad \begin{array}{l} \frac{\hbar\omega}{2E_0} + \frac{E}{E_0} \\ \frac{\hbar\omega}{E_0} + \frac{E}{E_0} \\ \frac{E}{E_0} \end{array} \right\} = 1 + \frac{\hbar\omega}{2E_0} - \frac{\hbar^2\omega^2}{48E_0^2}. \quad (52)$$

For the two-dimensional case in this limit we can write

$$\left. \begin{array}{l} s = 0 \\ s = 1 \\ s = -1 \end{array} \quad \begin{array}{l} \frac{\hbar\omega}{2E_0} + \frac{E}{E_0} \\ \frac{\hbar\omega}{E_0} + \frac{E}{E_0} \\ \frac{E}{E_0} \end{array} \right\} = 1 + \frac{\hbar\omega}{2E_0} - \frac{\hbar^2\omega^2}{24E_0^2}. \quad (53)$$

In the strong field the dependence on spin is not disappeared. In three-dimensional model energetic levels approach to specific values of spectra, which determine the binding states. For different values of particle spin the displacements levels of binding energy are at the equal distances with respect to boundaries of the continuous spectra. In this connection for the tree-dimensional case it can be represented in the form

$$\left. \begin{array}{l} s = 0 \\ s = 1 \\ s = -1 \end{array} \right\} \begin{array}{l} \frac{\hbar\omega}{2} - E' \\ \hbar\omega - E' \\ - E' \end{array} = \hbar\omega \left(0.295 + 0.452\sqrt{\frac{E_0}{\hbar\omega}} + 0.367\frac{E_0}{\hbar\omega} \right). \quad (54)$$

Hence for cases spinless particles $s = 0$ and for the electron with $s = 1$ in strong fields we have positive values of the binding energy levels whereas for electrons with $s = -1$ it remains negative.

It is ease to see that dependence on spin parameters in two-dimensional model can be written in the analogical form

$$\left. \begin{array}{l} s = 0 \\ s = 1 \\ s = -1 \end{array} \right\} \begin{array}{l} \frac{\hbar\omega}{2} - E' \\ \hbar\omega - E' \\ - E' \end{array} = \frac{\hbar\omega}{\ln \frac{\hbar\omega}{E_0}} + \frac{C \hbar\omega}{(\ln \frac{\hbar\omega}{E_0})^2} \quad (55)$$

From Eq.(55) one can see that in this limit the energy levels also is not dependent on the energy of the particle in the absence of the field. The distinctive feature of this case is the binding energy levels for the range of supper strong magnetic fields when $\ln(\hbar\omega/E_0) \gg 1$ are directly approaching to the boundaries of the continuous spectra for all considering spin values.

It is shown that shifts of energy levels of a polarized electron arising under the action of a weak magnetic field as in the three or two-dimensional models for different values of the particle spin go on by the similar way. We have the line displacements as the themselves levels for $s = 1$ and $s = -1$ and analogical shifts of the boundaries of continuous spectra for $s = 1$. The same picture we have and in the case of spinless particle where the line shift of the continuous spectrum boundary takes place. It is clear that in case of the weak intensity a magnetic field indeed plays a stabilizing role in considering systems, because the depth of the binding energy levels is increased under the field action independently on the particle spin. However in the strong field limit our results showered nonlinear dependence on the field intensity of the level displacements. Nevertheless in this limit the conditions

are determined the boundaries of continuous spectra as before have linear dependence on the field. Then in super strong magnetic fields one may find that binding energy levels approaching to the boundaries of continuous spectra. The existing distinctions can be formulated in the following way. In the model of (3+1) dimension there is limiting positions of energy levels, which are at fixed distances from boundaries of continuous spectra. In case of two-dimensional model energy levels in a super strong magnetic field asymptotically approach to boundaries of a continuous spectra. But in both cases we have increasing of instability of systems in strong magnetic fields. So this conclusion disproves the common opinion about a permanently stabilizing role of a magnetic field in ionization processes.

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